Reggio Calabria, September 2009

Permeability, tortuosity and attenuation of waves in porous materials

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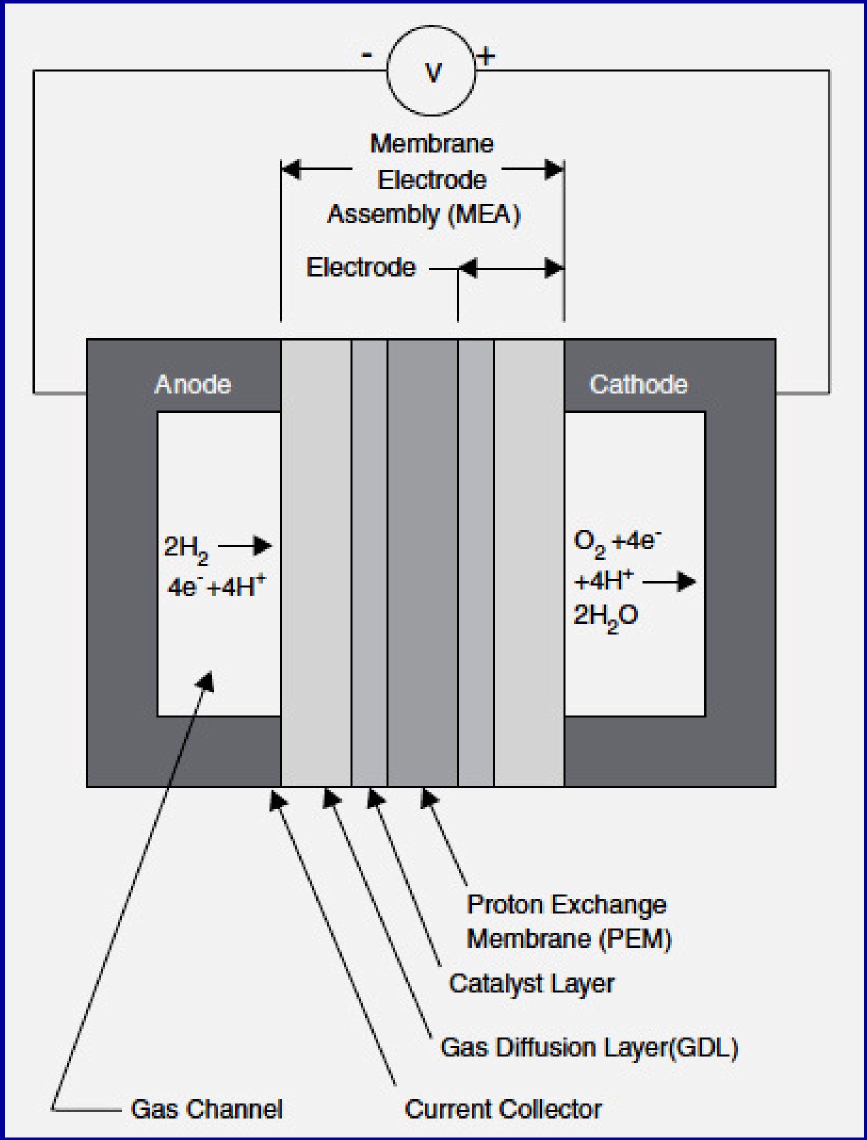
CONTENTS

Reference:

Wilmanski K.; *Permeability, tortuosity, and attenuation of waves in porous materials,* Civil and Environmental Engineering Reports (CEER), Zielona Gora, **5** (2010, to appear).

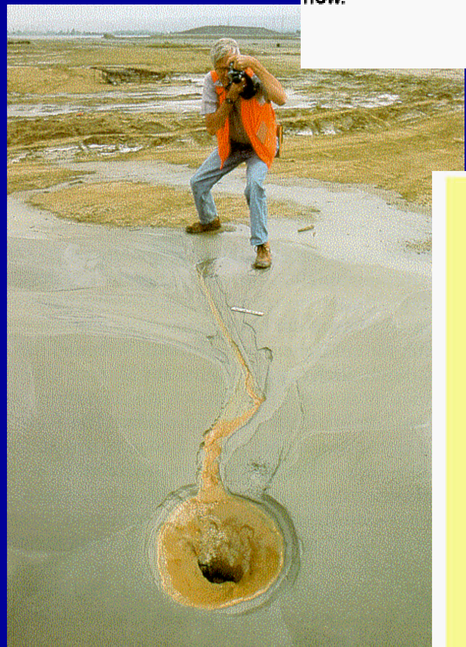
1. APPEARANCE OF DIFFUSION IN REAL POROUS SOLIDS

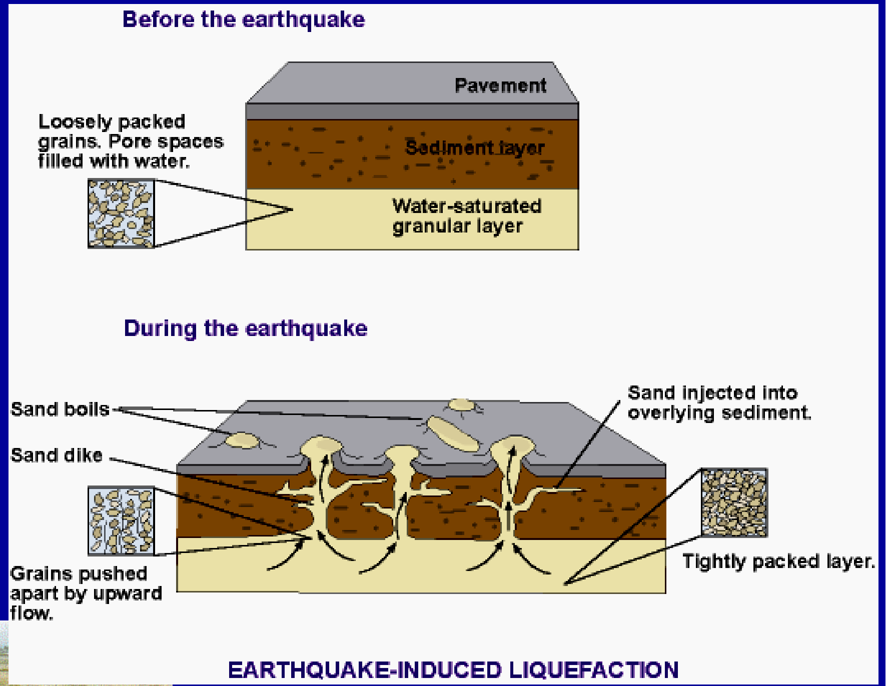
A. FUEL CELLS


-
V
+
Membrane
Electrode
Assembly (MEA)
Electrode
Anode
Cathode
2H2
4e- +4H+
O2 +4e-
+4H+
2H2O
Proton Exchange
Membrane (PEM)
Catalyst Layer
Gas Diffusion Layer(GDL)
Gas Channel
Current Collector

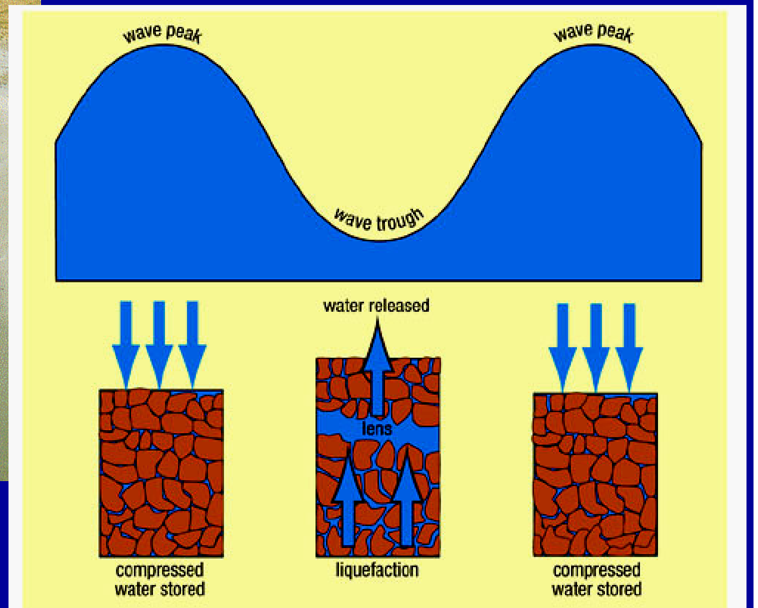
Palsas in Berlin – ice lenses (cryosuction)

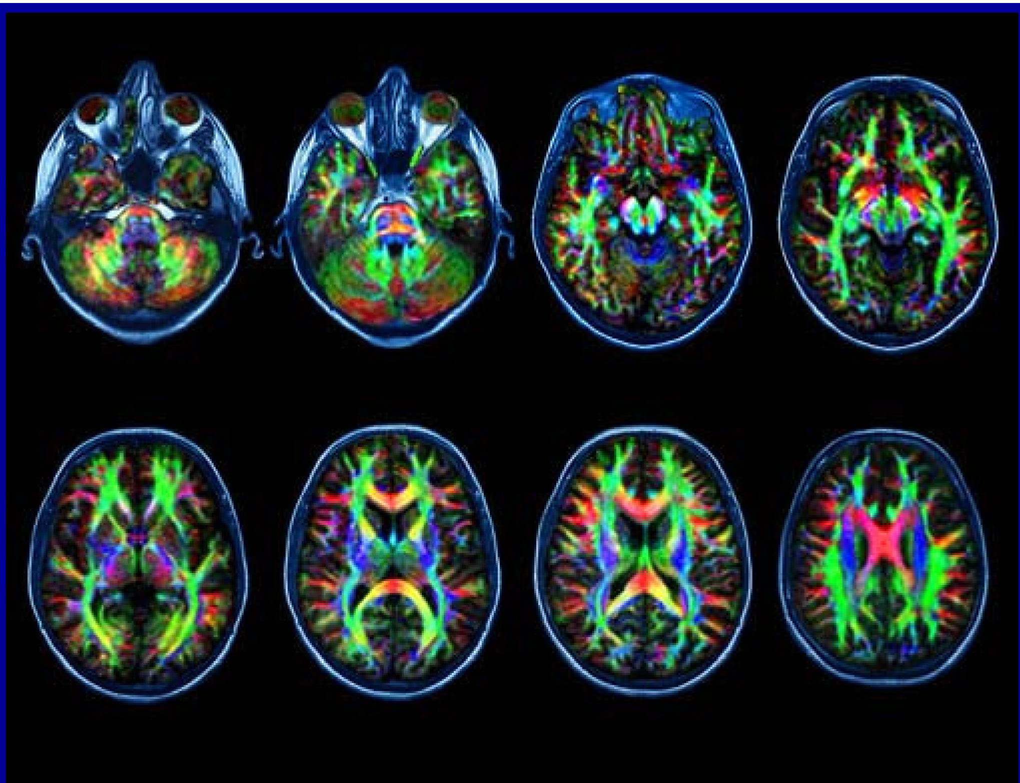
B. SOILS AND ROCKS

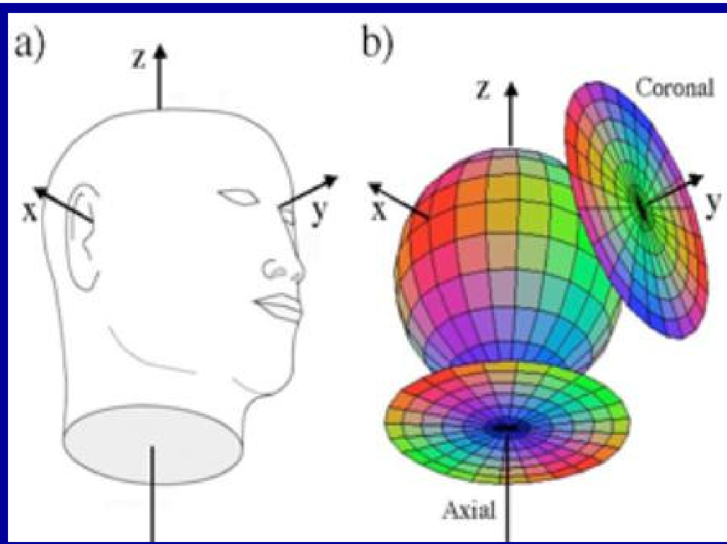




Liquefaction



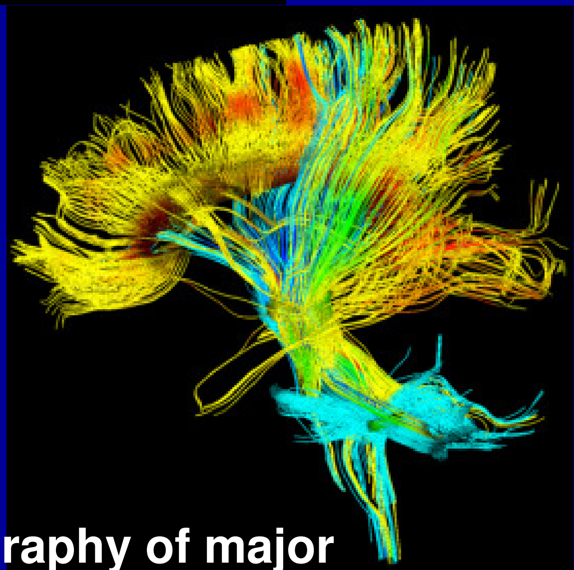




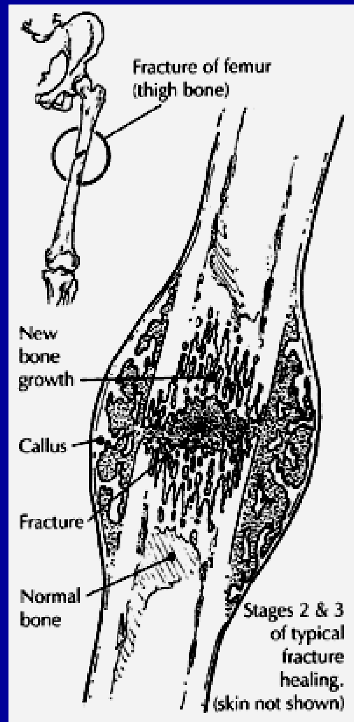
Relation between colors and directions

Diffusion MRI: anisotropy of permeability in brain

C. TISSUES

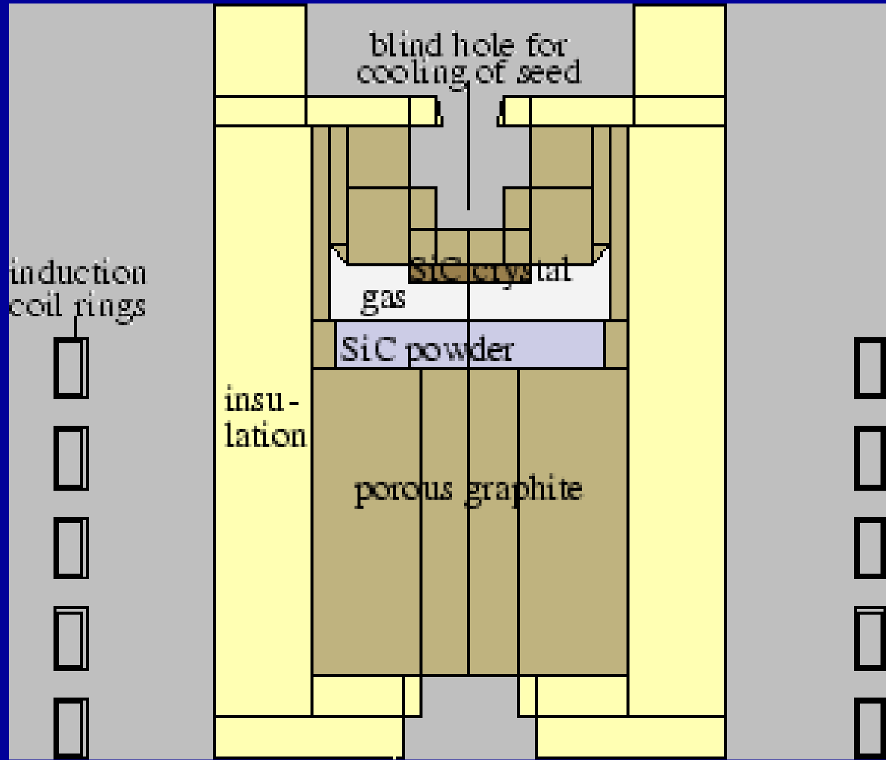


Diffusion MRI: tractography of major brain white matter tracts



Healing of the fructured bone

Pollen car filter



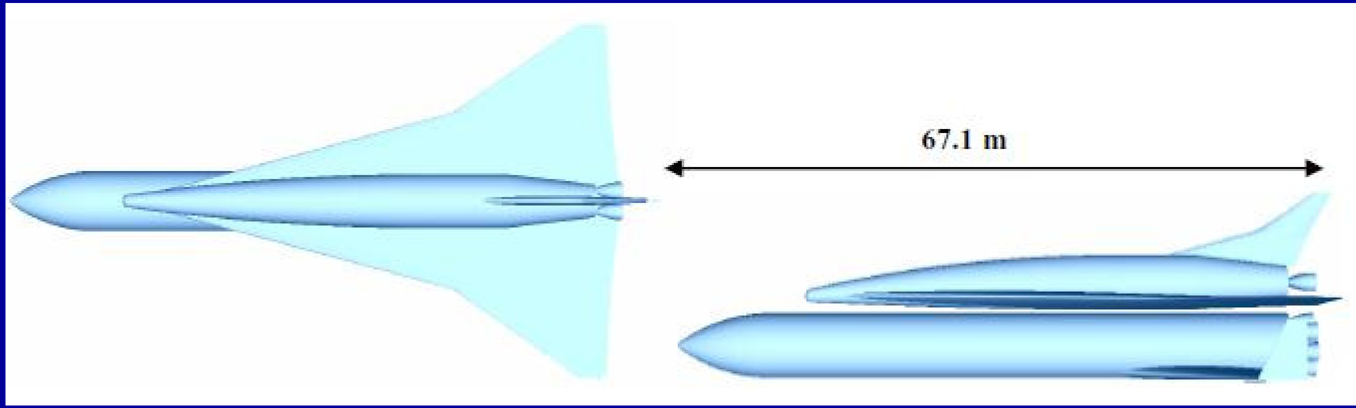
Schematic of growth of SiC-crystal

D. FILTERS AND TRANSPORT OF POLLUTANTS

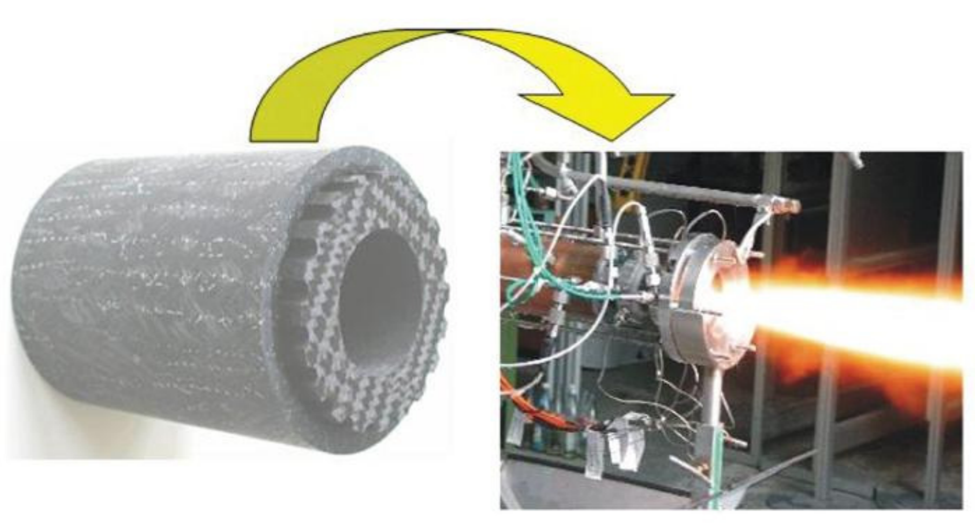
E. CRYSTAL GROWTH BY SUBLIMATION

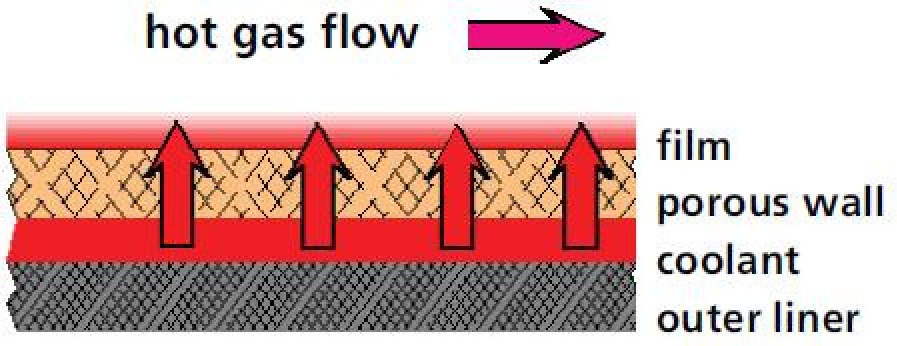
F. TRANSPIRATION COOLING

SpaceLiner



	GLOW Mass [kg]	Mass at burnout [kg]	Propellant mass [kg]	Fuselage length [m]	Max. fuselage diameter [m]	Wing span [m]	Projected wing surface area [m^2]
Orbiter	275,200	120,200	155,000	53	6	40	955
Booster	818,534	114,534	704,000	67.1	7	25.5	325
Total	1093,734	234,734	859,000	-	-	-	-





Ceramic Matrix Composite (CMC) in SpaceLiner Engine

2. LINEAR POROELASTIC MODEL OF SATURATED MATERIALS; ISOTHERMAL PROCESSES

Fields: $\left\{\rho^S, \rho^F, n, \mathbf{v}^S, \mathbf{v}^F, \mathbf{e}^S\right\},$

ρ^S, ρ^F - partial mass densities

$\mathbf{v}^S = v_i^S \mathbf{e}_i, \quad \mathbf{v}^F = v_i^F \mathbf{e}_i$ - partial velocities

$\mathbf{e}^S = e_{ij}^S \mathbf{e}_i \otimes \mathbf{e}_j$ - Almansi – Hamel deformation tensor

n - current porosity

Linearity conditions

$$\left\|\mathbf{e}^S\right\| << 1, \quad |\varepsilon| << 1,$$

$$\left(\mathbf{e}^S - \lambda^{(\alpha)}\mathbf{1}\right)\mathbf{n}^{(\alpha)} = \mathbf{0}, \quad \alpha = 1,2,3, \quad \left\|\mathbf{e}^S\right\| = \max\left|\lambda^{(\alpha)}\right|,$$

$$\varepsilon = \frac{\rho_0^F - \rho^F}{\rho_0^F},$$

Volume changes

$$e = tr\mathbf{e}^S = \sum_{\alpha=1}^{3} \lambda^{(\alpha)}.$$

Partial mass and momentum balance equations

$$\frac{\partial \rho^S}{\partial t} + \rho_0^S \operatorname{div} \mathbf{v}^S = 0, \quad \frac{\partial \rho^F}{\partial t} + \rho_0^F \operatorname{div} \mathbf{v}^F = 0,$$

$$\rho_0^S \frac{\partial \mathbf{v}^S}{\partial t} = \operatorname{div} \mathbf{T}^S + \hat{\mathbf{p}}^S + \rho_0^S \mathbf{b}^S,$$

$$\rho_0^F \frac{\partial \mathbf{v}^F}{\partial t} = \operatorname{div} \mathbf{T}^F + \hat{\mathbf{p}}^F + \rho_0^F \mathbf{b}^F,$$

$$\mathbf{T}^F = -p^F \mathbf{1} \quad \text{i.e.} \quad \sigma_{\mathrm{kk}}^{\mathrm{F}} = -3p^F,$$ (ideal fluid)

$$\hat{\mathbf{p}}^S = -\hat{\mathbf{p}}^F.$$ (conservation of momentum)

Porosity balance equation

Wilmanski K.: *A Thermodynamic Model of Compressible Porous Materials with the Balance Equation of Porosity,* Transp. Porous Media **32** (1998) 21-47**.**

$$\frac{\partial \Delta_n}{\partial t} + \Phi_0 \operatorname{div}\left(\mathbf{v}^F - \mathbf{v}^S\right) = \hat{n}, \quad \Delta_n = n - n_E.$$

Constitutive relations for isotropic materials

$$\mathbf{T}^S = \mathbf{T}_0^S + \lambda^S e\mathbf{1} + 2\mu^S \mathbf{e}^S + Q\varepsilon\mathbf{1} + \underline{\beta(n - n_E)\mathbf{1}} - N(n - n_0)\mathbf{1},$$

$$p^F = p_0^F - Qe - \rho_0^F \kappa\varepsilon + \underline{\beta(n - n_E)} + N(n - n_0),$$

$$\hat{\mathbf{p}}^S = -\hat{\mathbf{p}}^F = \pi * (\mathbf{v}^F - \mathbf{v}^S),$$

$$n_E = n_0(1 + \delta e),$$

$$\pi * (\mathbf{v}^F - \mathbf{v}^S) =$$

$$= \pi(0)(\mathbf{v}^F(t) - \mathbf{v}^S(t)) + \int_0^\infty \frac{d\pi}{ds}(s)\left[\mathbf{v}^F(t-s) - \mathbf{v}^S(t-s)\right]ds.$$

Wilmanski K.: *A few remarks on Biot's model and linear acoustics of poroelastic saturated materials,* Soil Dynamics & Earthquake Eng., **26,** 6-7 (2006) 509-536

Biot M. A.: *Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-Frequency Range*, J. Acoust. Soc. Am., **28,** (1956) 168-178**.**

Biot's model:

$$\beta = 0, \quad N = 0, \quad \hat{n} = 0.$$

Then porosity balance equation →

$$n = n_0\left(1 + \delta e + \frac{\Phi_0}{n_0}(\varepsilon - e)\right),$$

Influence of relative acceleration

$$\hat{\mathbf{p}}^S = \pi * \left(\mathbf{v}^F - \mathbf{v}^S\right) - \rho_{12}\mathbf{a}_r,$$

$$\hat{\mathbf{a}}_r = \underline{\frac{\partial}{\partial t}\left(\mathbf{v}^F - \mathbf{v}^S\right) + \left(\mathbf{v}^S \cdot \operatorname{grad}\right)}\left(\mathbf{v}^F - \mathbf{v}^S\right) -$$

$$-(1-\varsigma)\left(\left(\mathbf{v}^F - \mathbf{v}^S\right)\cdot \operatorname{grad}\right)\mathbf{v}^F - \varsigma\left(\left(\mathbf{v}^F - \mathbf{v}^S\right)\cdot \operatorname{grad}\right)\mathbf{v}^S,$$

$$\mathbf{T}^S = \mathbf{T}^S = \mathbf{T}_0^S + \lambda^S e\mathbf{1} + 2\mu^S\mathbf{e}^S + Q\varepsilon\mathbf{1} + \beta\left(n - n_E\right)\mathbf{1} -$$

$$- N\left(n - n_0\right)\mathbf{1} - \underline{\underline{\varsigma\rho_{12}\left(\mathbf{v}^F - \mathbf{v}^S\right)\otimes\left(\mathbf{v}^F - \mathbf{v}^S\right)}},$$

$$p^F = p_0^F - Qe - \rho_0^F\kappa\varepsilon + \beta\left(n - n_E\right) +$$

$$+ N\left(n - n_0\right) + \underline{\underline{\tfrac{1}{3}(1-\varsigma)\rho_{12}\left(\mathbf{v}^F - \mathbf{v}^S\right)\cdot\left(\mathbf{v}^F - \mathbf{v}^S\right)}}.$$

Wilmanski K.: *Tortuosity and objective relative accelerations in the theory of porous materials,* Proc. Roy. Soc. A, **461** (2005) 1533-1561

Field equations without coupling β and porosity sources

Fields

$$\{\mathbf{v}^S, \mathbf{v}^F, \mathbf{e}^S, \varepsilon\},$$

$$\rho_0^S \frac{\partial v_i^S}{\partial t} - \rho_{12}\left(\frac{\partial v_i^F}{\partial t} - \frac{\partial v_i^S}{\partial t}\right) = \lambda^S \frac{\partial \mathrm{e}}{\partial \mathrm{x}_\mathrm{i}} + 2\mu^S \frac{\partial \mathrm{e}_{\mathrm{ij}}^{\mathrm{S}}}{\partial \mathrm{x}_\mathrm{j}} +$$

$$+ Q\frac{\partial \varepsilon}{\partial x_i} - N\frac{\partial n}{\partial x_i} + \pi * \left(v_i^F - v_i^S\right) + \rho_0^S b_i^S,$$

$$\rho_0^F \frac{\partial v_i^F}{\partial t} + \rho_{12}\left(\frac{\partial v_i^F}{\partial t} - \frac{\partial v_i^S}{\partial t}\right) = \rho_0^F \kappa \frac{\partial \varepsilon}{\partial \mathrm{x}_\mathrm{i}} -$$

$$- Q\frac{\partial e}{\partial x_i} - N\frac{\partial n}{\partial x_i} - \pi * \left(v_i^F - v_i^S\right) + \rho_0^F b_i^F,$$

$$\frac{\partial e_{ij}^S}{\partial t} = \frac{1}{2}\left(\frac{\partial v_i^S}{\partial x_j} + \frac{\partial v_j^S}{\partial x_i}\right), \quad \frac{\partial \varepsilon}{\partial t} = \frac{\partial v_i^F}{\partial x_i},$$

$$n = n_0\left(1 + \delta e + \frac{\Phi_0}{n_0}(\varepsilon - e)\right).$$

Field equations without coupling β and porosity sources

$$\rho_0^S \frac{\partial v_i^S}{\partial t} - \rho_{12}\left(\frac{\partial v_i^F}{\partial t} - \frac{\partial v_i^S}{\partial t}\right) = \lambda^S \frac{\partial e}{\partial x_i} + 2\mu^S \frac{\partial e_{ij}^S}{\partial x_j} +$$

$$+ Q\frac{\partial \varepsilon}{\partial x_i} + N\frac{\partial n}{\partial x_i} + \pi * \left(v_i^F - v_i^S\right) + \rho_0^S b_i^S,$$

$$\rho_0^F \frac{\partial v_i^F}{\partial t} + \rho_{12}\left(\frac{\partial v_i^F}{\partial t} - \frac{\partial v_i^S}{\partial t}\right) = \rho_0^F \kappa \frac{\partial \varepsilon}{\partial x_i} -$$

$$- Q\frac{\partial e}{\partial x_i} - N\frac{\partial n}{\partial x_i} - \pi * \left(v_i^F - v_i^S\right) + \rho_0^F b_i^F,$$

Fields

$$\left\{\mathbf{v}^S, \mathbf{v}^F, \mathbf{e}^S, \varepsilon\right\},$$

These parameters follow from micro- macro transition

$$\frac{\partial e_{ij}^S}{\partial t} = \frac{1}{2}\left(\frac{\partial v_i^S}{\partial x_j} + \frac{\partial v_j^S}{\partial x_i}\right), \quad \frac{\partial \varepsilon}{\partial t} = \frac{\partial v_i^F}{\partial x_i},$$

$$n = n_0\left(1 + \delta e + \frac{\Phi_0}{n_0}(\varepsilon - e)\right).$$

Wilmanski K.: *On Microstructural Tests for Poroelastic Materials and Corresponding Gassman-type Relations,* Geotechnique, **54,** 9 (2004) 593-603

Remark: Fallacy of variational formulation of Biot's model

increment of fluid content instead of volume changes of the fluid

$$\zeta = n_0 \left(\varepsilon - e \right),$$

Then

$$\frac{\partial \zeta}{\partial t} = n_0 \frac{\partial}{\partial t} \left(\varepsilon - e \right) = \operatorname{div} \left(\mathbf{v}^F - \mathbf{v}^S \right),$$

and the dissipation (second law of thermodynamics)

$$\mathcal{D} = \left(\mathbf{v}^F - \mathbf{v}^S \right) \cdot \left[\pi * \left(\mathbf{v}^F - \mathbf{v}^S \right) \right] \geq 0,$$

ρ_{12} **does not contribute!!**

hence, the increment of fluid content measures deviations from thermodynamic equilibrium. Consequently, there exists no variational principle for Biot's model!

Porosity balance has in this variable the form of evolution equation –
- no boundary conditions needed!

$$\frac{\partial}{\partial t} \left(\Delta_n - \frac{\Phi_0}{n_0} \zeta \right) = \hat{n},$$

3. STRUCTURE OF MOMENTUM SOURCE; PERMEABILITY, TORTUOSITY

REMINDER: Momentum balance for the fluid without inertial forces

$$\operatorname{grad} p^F + \pi\left(\mathbf{v}^F - \mathbf{v}^S\right) - \rho_0^F \mathbf{b}^F = 0, \quad \mathbf{b}^F = \gamma n_0 z \mathbf{e}_Z$$

with the body force in direction of z-axis

Integration for the inclined column:

$$q = \left|\mathbf{v}^F - \mathbf{v}^S\right| = K\frac{\varphi_1 - \varphi_2}{L},$$

$$\varphi_1 = h_1 + \frac{p_1^F}{\gamma n_0}, \quad \varphi_2 = h_2 + \frac{p_2^F}{\gamma n_0},$$

$$K = \frac{n_0 \gamma}{\pi}, \quad \gamma = \frac{g \rho_0^F}{n_0},$$

- Darcy‘s relation!

K – hydraulic conductivity, related to the true dynamic viscosity of the fluid

$$K = \frac{\kappa_p \gamma}{\mu_v} \quad \Rightarrow \quad \kappa_p = \frac{n_0 \mu_v}{\pi},$$

κ_p **- intrinsic permeability.**

Examples of hydraulic conductivity, intrinsic permeability and permeability coefficient for a porous material saturated with water (normal conditions)

soil	K [m/s]	κ_p [darcy]=10^{-12}[m^2]	π [kg/m^3s]
well sorted gravel	$1 – 10^{-3}$	10^5 - 10^2	$10^3 – 10^6$
oil reservoir	10^{-4} - 10^{-6}	10 - 10^{-1}	10^7 - 10^9
sandstone	10^{-7} - 10^{-8}	$10^{-2} – 10^{-3}$	$10^{10} – 10^{11}$
granite	10^{-11} - 10^{-12}	10^{-6} - 10^{-7}	$10^{14} – 10^{15}$

Necessary extensions of the permeability:

1. **Tortuosity**
2. **Anisotropy**
3. **Hereditary**
4. **Nonlinearity**

Blake-Kozeny-Epstein relation:

$$K = \frac{D_h^2 n_0}{b\mu_v \tau^2},$$

b – capillary shape factor (32 for circular pores, 48 for parallel slits)

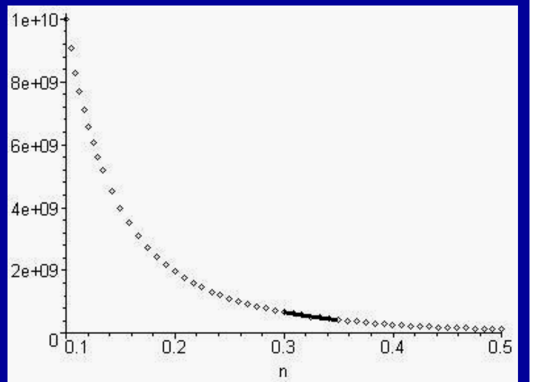
D_h – hydrolic diameter; e.g. N spheres of radius d in REV:

$$V_f = \frac{1}{6}\pi d^3 N \frac{n_0}{1-n_0}, \quad S_s = \pi d^2 N \quad \Rightarrow$$

$$\Rightarrow \quad D_h = 4\frac{V_f}{S_s} = \frac{2}{3} d \frac{n_0}{1-n_0}.$$

Then

$$K = \frac{4d^2}{9b\mu_v \tau^2} \frac{n_0^3}{(1-n_0)^2} \quad \Rightarrow \quad \pi = \frac{9}{4} \frac{\gamma b \mu_v}{d^2} \tau^2 \left(\frac{1-n_0}{n_0}\right)^2,$$



τ - tortuosity, i.e. ratio of the length of a streamline between two points to their distance, $\tau > 1$

Anisotropic diffusion

Bear J., Bachmat Y.: *Introduction to Modeling of Transport Phenomena in Porous Media*, Dordrecht, Kluwer Academic Publishers 1991

Fluid „discharge“ (relative velocity) for tensorial permeability

$$v_i^F - v_i^S = -\frac{k_{ij}}{n_0 \mu_v}\left(\frac{\partial p^F}{\partial x_j} + \rho^F g \frac{\partial z}{\partial x_j}\right),$$

where symmetric permeability tensor has the structure

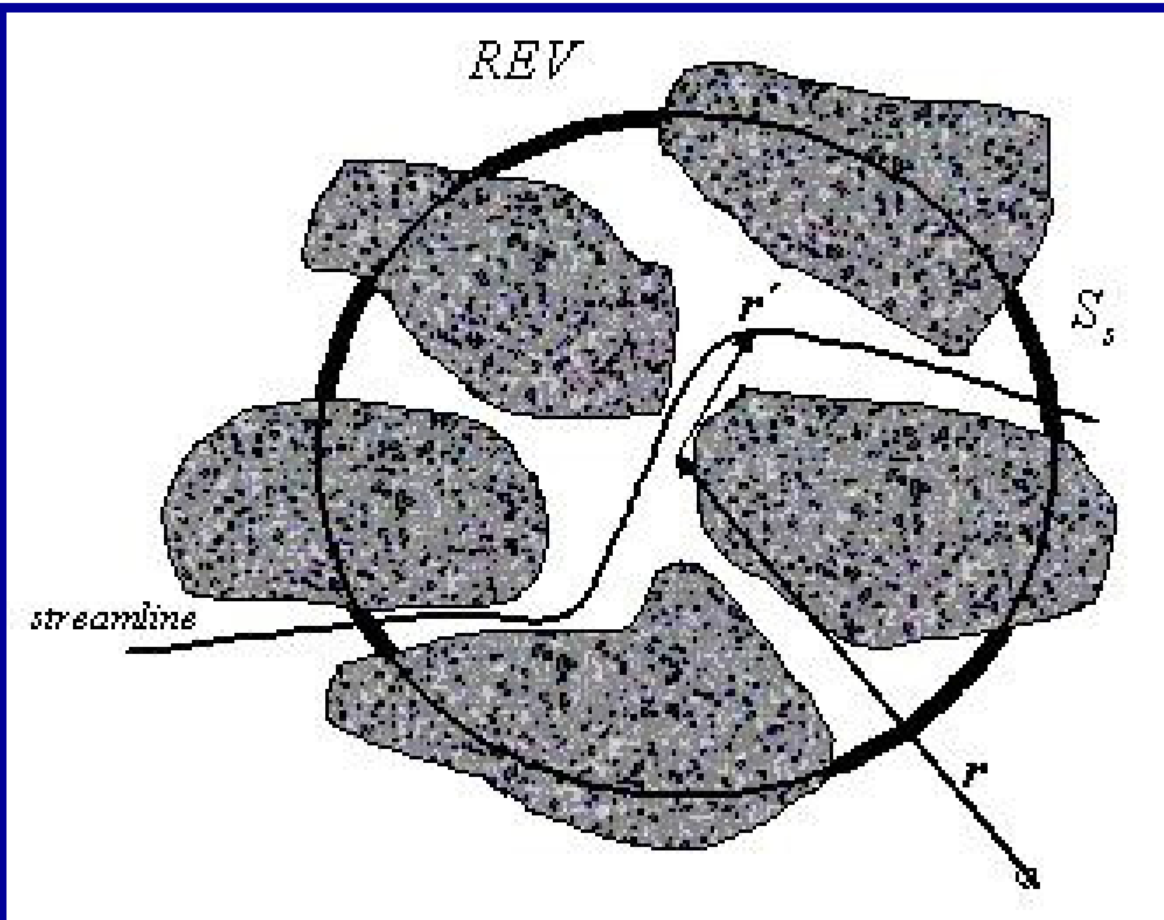
$$k_{ij} = B T_{ij},$$

T_{ij} **- symmetric tortuosity tensor defined as the surface average (static moment) of the** REV **– boundary intersected by streamlines**

B **- conductance given by the relation** $B = \dfrac{D_h^2 n_0}{b\gamma}$ or $B = \kappa_p \tau^2.$

$$\mathbf{r} = x_i \mathbf{e}_i, \quad \mathbf{r}' = \xi_i \mathbf{e}_i,$$

$$\mathbf{r}' = \mathbf{r}'(s), \quad \mathbf{n} = \frac{d\xi_i}{ds}\mathbf{e}_i,$$



Schematic of the Representative Elementary Volume (*REV*) with a streamline intersecting *REV*-boundary at a point of the surface S_s. The latter is indicated by the thick line.

For *REV* – sphere of radius *R*

$$\mathbf{T} = T_{ij}\mathbf{e}_i \otimes \mathbf{e}_j = \frac{1}{n_0 V}\int_{S_s} R\,\mathbf{n} \otimes \mathbf{n}\, dS$$

Spectral representation

$$\mathbf{T} = \sum_{\alpha=1}^{3} T^{(\alpha)} \mathbf{t}^{(\alpha)} \otimes \mathbf{t}^{(\alpha)},$$

For isotropic diffusion properties (J. Bear)

$$T^{(1)} = T^{(2)} = T^{(3)} = \frac{1}{\tau^2}$$

Source of momentum for anisotropic media with dependence on relative acceleration

$$\hat{p}_i^S = \pi_0 T_{ij}^{-1}\left(v_j^F - v_j^S\right) - \rho_{12} a_{ri}, \quad \mathbf{a}_r = a_{ri}\mathbf{e}_i,$$

$$\pi_0 = \frac{n_0 \mu_v}{B}.$$

M. A. Biot argument for necessity of added mass:

$$\rho_{12}\frac{\partial v_x^S}{\partial t} + Q_x^F = 0, \quad Q_x^F = \rho_0^F \kappa \frac{\partial \varepsilon}{\partial x} - Q\frac{\partial e}{\partial x} - \\ -\pi * \left(v_x^F - v_x^S\right) + \rho_0^F b_x^F,$$

"…the equation shows that when the solid is accelerated a force Q_x^F must be exerted on the fluid to prevent an average displacement of the latter".

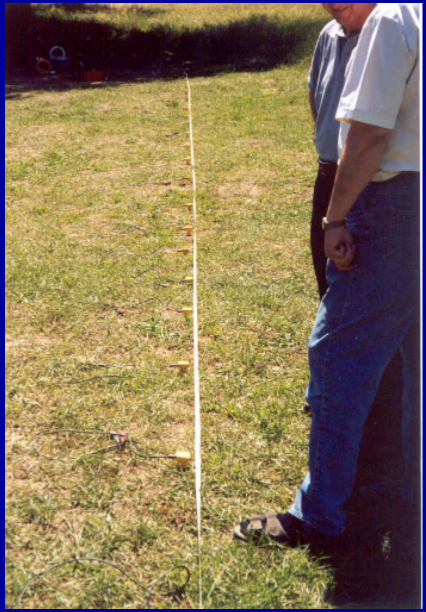
This is obviously not necessary. Hence, tortuosity may enter solely through permeability

4. MEASUREMENTS OF DIFFUSION PROPERTIES; NMR

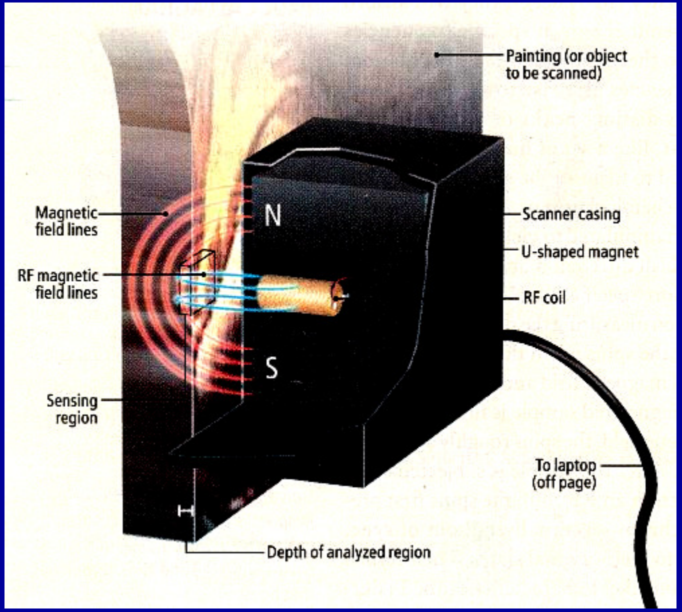
comp. G. Dagan this afternoon!

EXPERIMENTAL TECHNIQUES

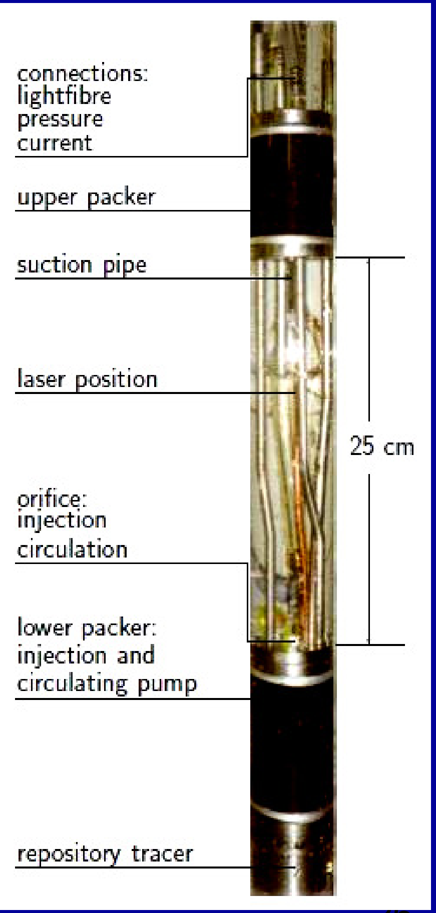
1. Mechanical flow velocimeters – permeability
2. Electric resistivity (Nernst-Einstein relation between diffusivity and conductance)
3. Diffusion Magnetic Resonance Imaging
4. Damping of acoustic waves - permeability
5. Surface waves (SASW) - porosity



SASW



Miniaturized surface NMR



Groundwater flow velocimeter

NMR Primer

In quantum mechanics description proton (spin-1/2 particle) possesses

- **intrinsic magnetic moment** $\boldsymbol{\mu}_p$
- **intrinsic angular momentum** $\mathbf{J}_p$

related to each other

$$\boldsymbol{\mu}_p = \gamma_p \mathbf{J}_p,$$

$$\gamma_P = 2\pi \times 42.5764 \times 10^6 \ \frac{\text{rad}}{\text{Tesla}}$$ **- gyromagnetic constant**

Wave function for proton in magnetic field satisfies Schrödinger equation

$$\hbar \frac{\partial \psi}{\partial t} = i\mathbf{B} \cdot \boldsymbol{\mu}_P \psi,$$

This yields the following equation for the average macroscopic intrinsic magnetic moment („observable“)

$$\frac{d\langle \boldsymbol{\mu}_P \rangle}{d\,t} = \gamma_P \langle \boldsymbol{\mu}_P \rangle \times \mathbf{B}.$$

For the field $\mathbf{B} = \mathbf{B}_0 = b_0 \mathbf{e}_3$

$$\frac{d\langle \mu_P \rangle_1}{dt} = \gamma_P b_0 \langle \mu_P \rangle_2, \quad \frac{d\langle \mu_P \rangle_2}{dt} = -\gamma_P b_0 \langle \mu_P \rangle_1,$$

$\langle \mu_P \rangle_3$ - constant

Solution – precession with the Larmor frequency: $\exp(-i\omega_0 t),\ \omega_0 = \gamma_P b_0.$

It has the order of 100 MHz for the field 1 Tesla („radio frequency“ range –RF)

Macroscopic model of many interacting particles with spin:

macroscopic magnetization in a magnetic field

$$\mathbf{M}_0(\mathbf{r}) = \frac{C}{T}\rho(\mathbf{r})\mathbf{B}_0(\mathbf{r}),$$

T – absolute temperature
ρ - mass density

For the field

$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0(\mathbf{r}) + \tilde{\mathbf{B}}(\mathbf{r},t), \quad |\mathbf{B}_0(\mathbf{r})| >> |\tilde{\mathbf{B}}(\mathbf{r},t)|,$$

Bloch (phenomenological) equation:

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} - \frac{1}{T_2}\mathbf{M}^{\perp} - \frac{1}{T_1}\left(\mathbf{M}^{\|} - \mathbf{M}_0\right),$$

$$\mathbf{M} = \mathbf{M}^{\perp} + \mathbf{M}^{\|},$$

$\mathbf{M}^{\perp}$: perpendicular to $\mathbf{B}_0$
$\mathbf{M}^{\|}$: parallel to $\mathbf{B}_0$

T_1, T_2 – relaxation times

Influence of diffusion – Bloch-Torrey equation

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} - \frac{1}{T_2}\mathbf{M}^{\perp} - \frac{1}{T_1}\left(\mathbf{M}^{\parallel} - \mathbf{M}_0\right) + \mathrm{div}\left(\mathbf{D}\,\mathrm{grad}\,\mathbf{M}\right),$$

$$\mathbf{D} = D_{ij}\mathbf{e}_i \otimes \mathbf{e}_j,$$

diffusivity tensor

Basic principle – measurements of relaxation times for different directions of the magnetic field

Example 1: Results of Xenon NMR measurements for some rocks

Rock Sample	Permeability κ_p [mD]; π [kg/m^{3}s]	Tortuosity	Effective Porosity	Absolute Porosity (pycnometer)
Fontainebleau	559 +/- 93; 1.53-2.15 x 10^8	3.45	0.113 +/- 0.007	0.125
Bentheimer	123 +/- 24; 10.1-6.8 x 10^8	NA	0.112 +/- 0.012	NA
Edwards Limestone	7.0 +/- 0.9; 1.27-1.63 x 10^{10}	4.76	0.151 +/- 0.011	0.233
Austin Chalk	2.6 +/- 0.3; 3.44-4.35 x 10^{10}	5.58	0.184 +/- 0.9	0.297
Cutbank H	0.64 +/- 0.1; 1.35-1.65 x 10^{11}	NA	0.0603 +/- 0.004	NA
Indiana Limestone	0.18 +/- 0.03; 4.76-6.67 x 10^{11}	7.69	0.071 +/- 0.006	NA

Example 2: Some results for core plugs of Alermoehe sandstone from various depths

Sample	Permeability (gas) κ_p π	Tortuosity	Porosity (pycnometer)
3224.45 [m]	0.16; 6.25×10^{11}	1.06	0.02
3235.34 [m]	11.6; 8.62×10^{9}	5.04	0.09
3236.79 [m]	3.59; 2.79×10^{10}	5.36	0.08
3240.69 [m]	20.7; 4.83×10^{9}	3.8	0.11
3241.44 [m]	3.13; 3.19×10^{10}	6.12	0.09

5. MONOCHROMATIC ACOUSTIC WAVES; SPEEDS AND ATTENUATION

Governing equations

$$\rho_0^S \frac{\partial v_i^S}{\partial t} - \rho_{12}\left(\frac{\partial v_i^F}{\partial t} - \frac{\partial v_i^S}{\partial t}\right) = \lambda^S \frac{\partial \mathrm{e}}{\partial \mathrm{x}_\mathrm{i}} + 2\mu^S \frac{\partial \mathrm{e}_\mathrm{ij}^\mathrm{S}}{\partial \mathrm{x}_\mathrm{j}} +$$

$$+ Q\frac{\partial \varepsilon}{\partial x_i} - N\frac{\partial n}{\partial x_i} + \pi * \left(v_i^F - v_i^S\right) + \rho_0^S b_i^S,$$

$$\rho_0^F \frac{\partial v_i^F}{\partial t} + \rho_{12}\left(\frac{\partial v_i^F}{\partial t} - \frac{\partial v_i^S}{\partial t}\right) = \rho_0^F \kappa \frac{\partial \varepsilon}{\partial \mathrm{x}_\mathrm{i}} -$$

$$- Q\frac{\partial e}{\partial x_i} - N\frac{\partial n}{\partial x_i} - \pi * \left(v_i^F - v_i^S\right) + \rho_0^F b_i^F,$$

$$\frac{\partial e_{ij}^S}{\partial t} = \frac{1}{2}\left(\frac{\partial v_i^S}{\partial x_j} + \frac{\partial v_j^S}{\partial x_i}\right), \quad \frac{\partial \varepsilon}{\partial t} = \frac{\partial v_i^F}{\partial x_i},$$

$$n = n_0\left(1 + \delta e + \frac{\Phi_0}{n_0}(\varepsilon - e)\right).$$

immaterial

Biot argued that the analysis of a flow of viscous fluid in channels of a porous material yields a dependence of the permeability on the frequency of a monochromatic wave. After inversion of the Fourier transform the following relation should follow for isotropic materials

$$\pi(t) = \pi_0 \tau^2 F(t),$$

Biot M. A.: *Theory of propagation of elastic waves in a fluid-saturated porous solid. II. Higher Frequency Range*, J. Acoust. Soc. Am., **28,** 2 (1956) 179-191**.**

where the dimensionless function F **depends on the frequency** ω in **the following way**

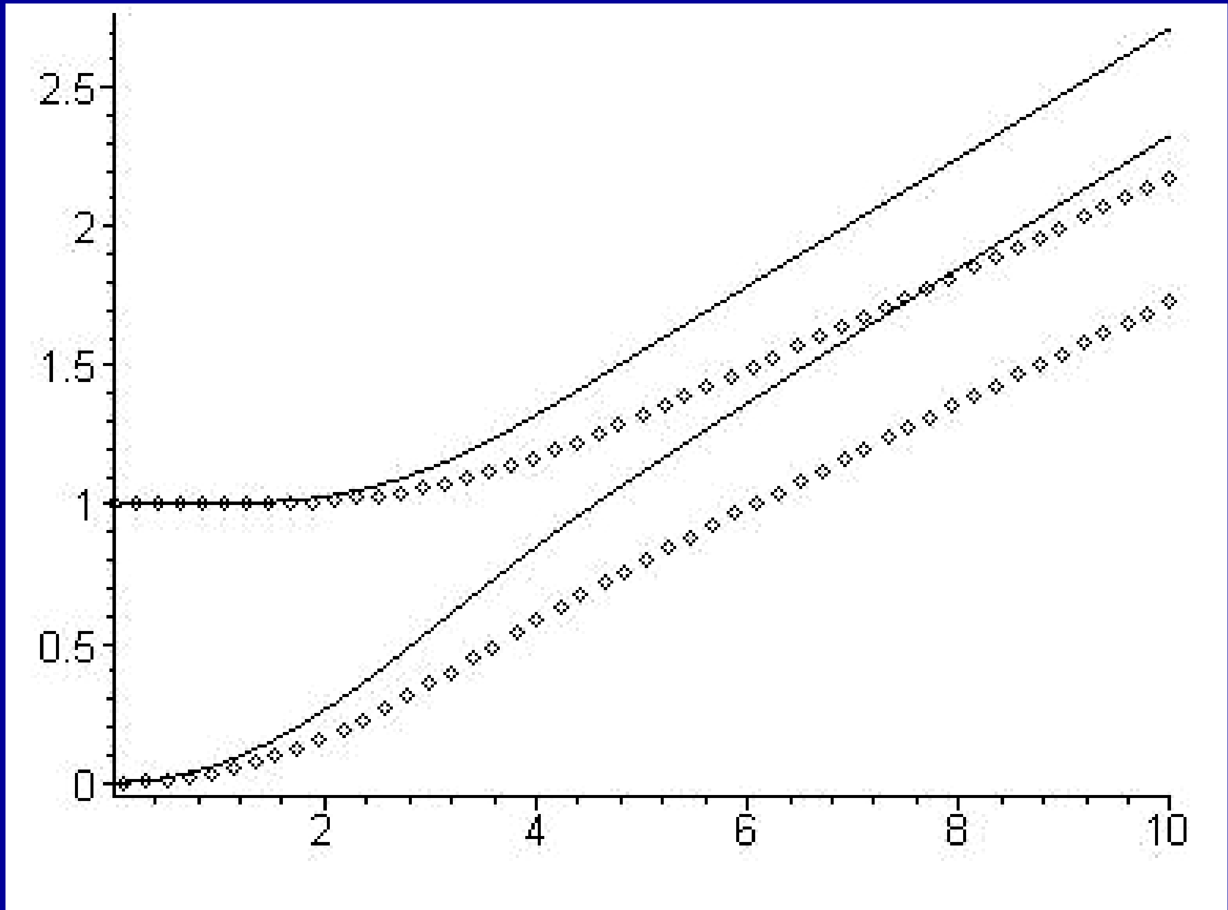
$$(1) \qquad F(\omega) = \frac{1}{3}\frac{\sqrt{i}\xi \tanh\left(\sqrt{i}\xi\right)}{1 - \frac{1}{\xi\sqrt{i}}\tanh\left(\sqrt{i}\xi\right)}, \qquad \xi = a\sqrt{\frac{\omega\rho}{\mu_v}},$$

for the flow between parallel walls

$$(2) \qquad F(\omega) = \frac{1}{4}\frac{\xi \mathrm{T}(\xi)}{1 - \frac{2}{i\xi}\mathrm{T}(\xi)}, \qquad \mathrm{T}(\xi) = \frac{\frac{d}{d\xi}\left(J_0\left(i\sqrt{i}\xi\right)\right)}{J_0\left(i\sqrt{i}\xi\right)},$$

in a circular duct

with $$F(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{i\omega t}\,d\omega,$$



Real (upper curves) and imaginary (lower curves) parts of functions $F(\xi)$

Solid lines correspond to the case of parallel walls and dotted lines to the circular duct

$\xi=1$ **corresponds for water to app. 1 kHz frequency**

Fallacy of relation between tortuosity τ and added mass ρ_{12}

Quotation from: Johnson D. L., Koplik J., Dashen R.: *Theory of dynamic permeability and tortuosity in fluid-saturated porous media*, J. Fluid Mech. **176** (1987) 379-402.

"Under the stated assumptions $\mathbf{v}$ is obviously linearly related to the pressure gradient at any frequency

$$\tilde{\alpha}(\omega)\rho_f \frac{\partial \mathbf{v}}{\partial t} = -\nabla P, \quad \phi \mathbf{v} = -\frac{\tilde{k}(\omega)}{\eta}\nabla P. \quad (2.1a,b),$$

($\eta = \mu_v, \phi = n$ in the notation of this lecture). The frequency-dependent tortuosity $\tilde{\alpha}(\omega)$ is defined in (2.1a) by analogy with the response of an ideal (nonviscous) fluid. … The frequency–dependent permeability $\tilde{k}(\omega)$ is defined in (2.1b) by analogy with the steady-state $(\omega = 0)$ definition.

$$\tilde{\alpha}(\omega) = \frac{i\eta\phi}{\tilde{k}(\omega)\omega\rho_f}. \quad (2.1c).$$

$$\ldots\tilde{\rho}_{12}(\omega) = -[\tilde{\alpha}(\omega) - 1]\phi\rho_f \quad (4.1b)"$$

(?!)

$$\frac{1}{\tilde{k}(\omega)} \sim \pi(\omega)$$

The above argument is physically and mathematically wrong. Physically, both equations (2.1) follow from <u>different</u> simplifications of momentum balance – comparison of apples and oranges. Mathematically, the first relation is hyperbolic, the second – parabolic.

Albers B., Wilmanski K.: *On modeling acoustic waves in saturated poroelastic media*, J. Engng. Mech., **131,** 9 (2005) 974-985.

Added mass has only a little influence on propagation of acoustic waves $\rho_{12} = 0.$

Monochromatic waves

$$v_i^S = V_i^S \mathcal{E}, \quad v_i^F = V_i^F \mathcal{E}, \quad e_{ij}^S = E_{ij}^S \mathcal{E}, \quad \varepsilon = E^F \mathcal{E},$$
$$\mathcal{E} = \exp\left[i\left(k_j x_j - \omega t\right)\right].$$

Propagation conditions and dispersion relations

$$\left[\left(\omega^2 + i\frac{\pi\omega}{\rho_0^S}\right)\delta_{ij} + \frac{\lambda^S}{\rho_0^S} k_i k_j + \frac{\mu^S}{\rho_0^S}\left(k^2 \delta_{ij} + k_i k_j\right)\right] V_j^S - \left(\frac{Q}{\rho_0^S} k_i k_j + i\frac{\pi\omega}{\rho_0^S}\delta_{ij}\right) V_j^F = 0,$$

$$\left(\frac{Q}{\rho_0^F} k_i k_j - i\frac{\pi\omega}{\rho_0^F}\delta_{ij}\right) V_j^S + \left[\left(\omega^2 + i\frac{\pi\omega}{\rho_0^F}\right)\delta_{ij} + \kappa k_i k_j\right] V_j^F = 0.$$

The solution of this eigenvalue problem yields two longitudinal waves: P1 and P2 (slow, Biot, second sound), and the transversal (shear) wave.

Relation for the wave number in the case of transversal waves

$$k^2 =$$

$$= \omega^2 \left[\omega^2 + i\omega \frac{\pi}{\rho_0^S} F(\omega) \left(1 + \frac{1}{r} \right) \right] / \left[c_0^{S\,2} \left(\omega^2 + i \frac{\pi}{\rho_0^S} F(\omega) \frac{1}{r} \right) \right],$$

$$r = \frac{\rho_0^F}{\rho_0^S}, \quad c_0^{S\,2} = \frac{\mu^S}{\rho_0^S}.$$

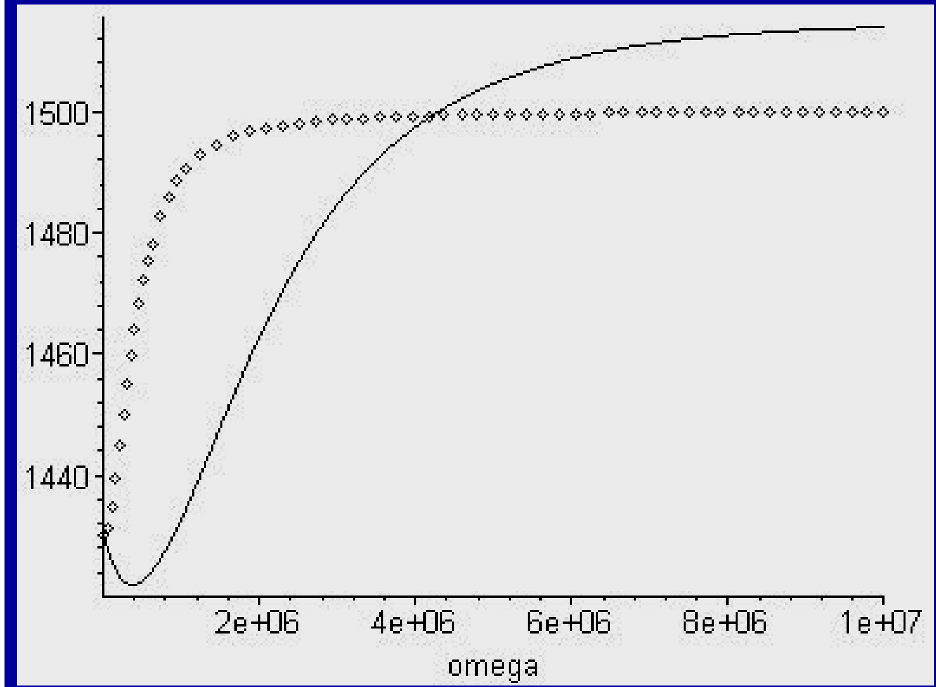
Numerical example for the data

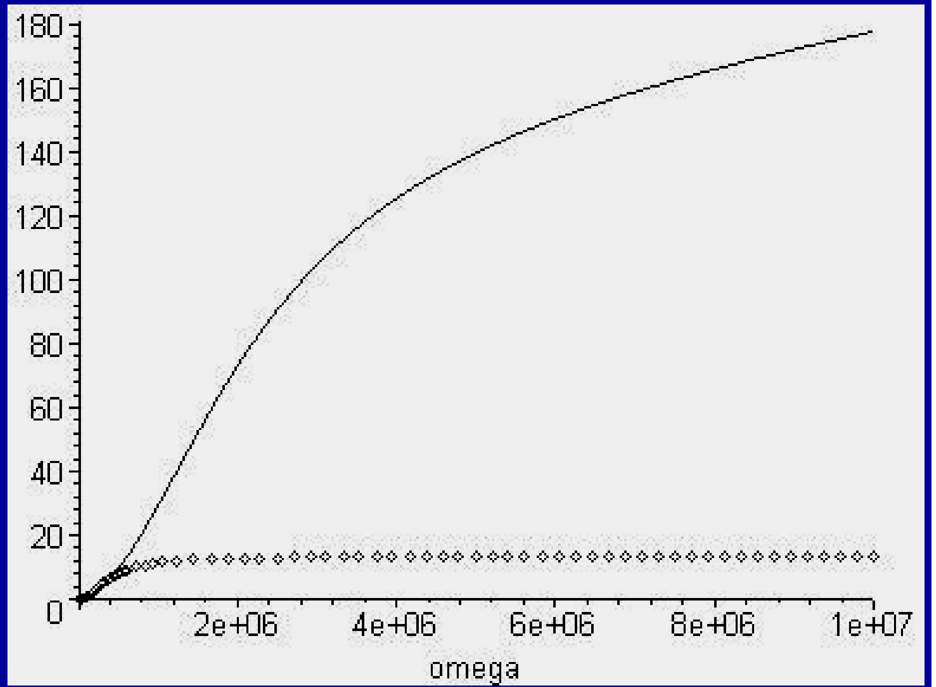
$$\rho_0^S = 2500 \left[\mathrm{kg/m^3}\right], \quad r := \frac{\rho_0^F}{\rho_0^S} = 0.1,$$

$$c_0^S = \frac{\mu^S}{\rho_0^S} = 1500 \left[\mathrm{m/s}\right], \quad \pi_0 = 10^8 \left[\mathrm{kg/m^3 s}\right],$$

$$a = 10^{-5} \left[\mathrm{m}\right], \quad \mu_v = 1.002 \times 10^{-3} \left[\mathrm{kg/m \cdot s}\right].$$

$\tau = 1:$

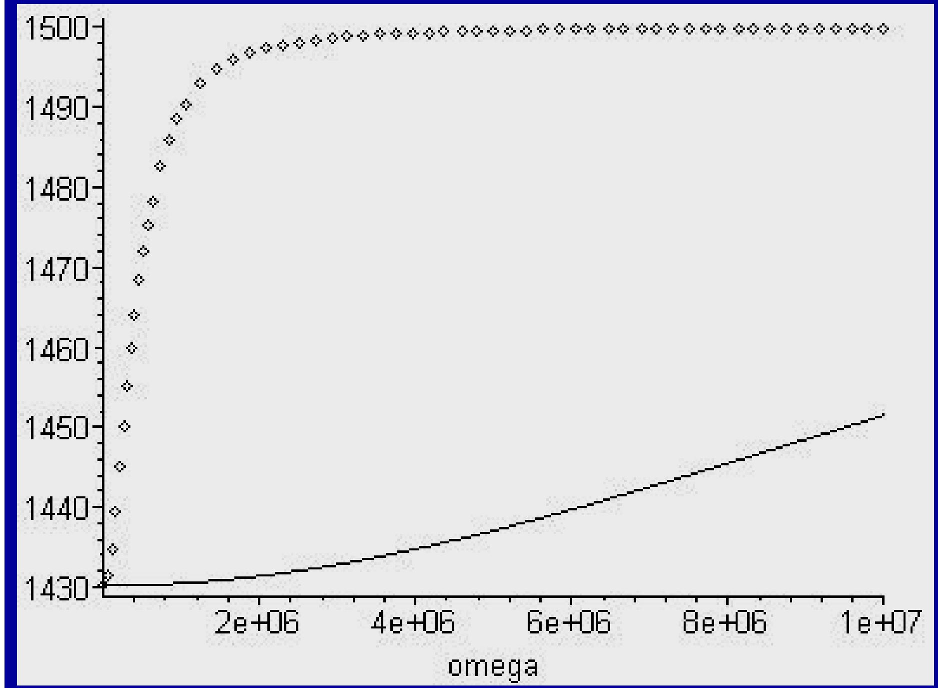




Phase speed $c = \omega / \mathrm{Re}\, k$ **of the shear wave; the dotted line corresponds to** $F=1$**, the solid line for** F **introduced by Biot**

Attenuation $\mathrm{Im}\, k$ **of the shear wave; the limit for the solid line is infinite**

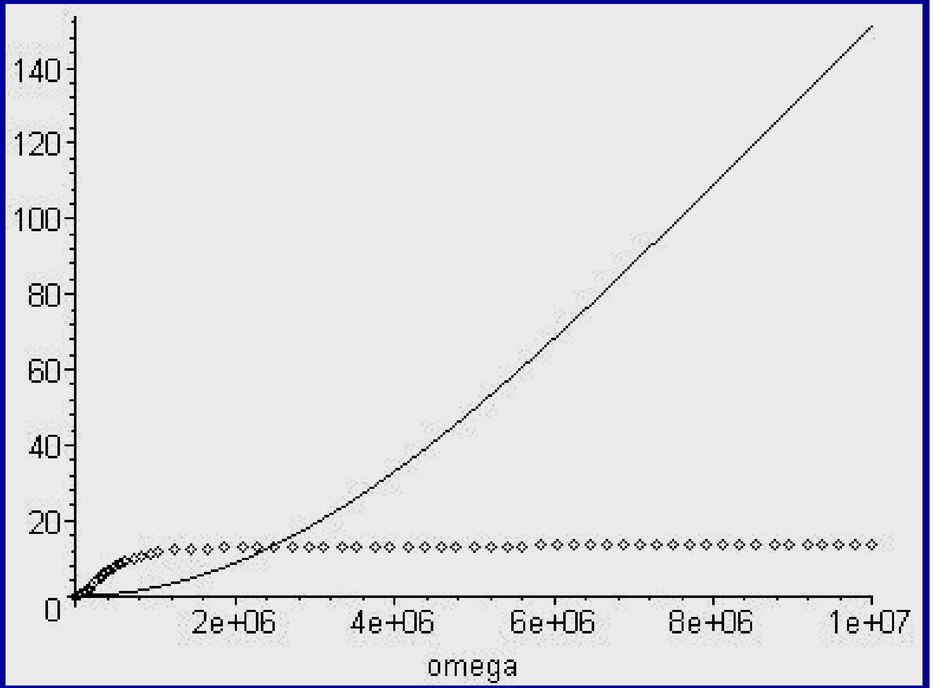
$F(\omega) = 1:$



Phase speed of the shear wave for two values of tortuosity:

$\tau = 1$ - dotted line

$\tau = 6$ - solid line



Attenuation of the shear wave for two values of tortuosity:

$\tau = 1$ - dotted line

$\tau = 6$ - solid line

the limit for the solid line is finite

6. CONCLUDING REMARKS; NONISOTHERMAL PROCESSES; NONLINEARITIES

1. Nonisothermal processes do not yield essential changes in permeability properties for linear models of fully saturated materials. Otherwise couplings with temperature changes influence the degree of saturation, surface properties of channels, capillary forces, etc.

2. Nonlinearities are essential, particularly for large Reynolds numbers, changes in microstructure (piping), liquefaction. The simplest (quadratic) correction was proposed by P. Forchheimer, many other models are applied as well.

3. Both nonlinearities and the nonisothermal character play an important role in the range of low temperatures (freezing and cryosuction) as well as in the range of high temperatures (melting, boundary layers). Little has been done.

Thank you for your attention!